Weinberg
2.4

$$\hat{P}^\mu U(\Lambda)\Psi = \Lambda^\mu_\rho p^\rho \, U(\Lambda)\Psi \qquad \text{(Weinberg 2.5.2)}$$

$$\Rightarrow \hat{P}_\mu \hat{P}^\mu U(\Lambda)\Psi = \Lambda^\mu_\rho p^\rho \, \hat{P}_\mu U(\Lambda)\Psi$$

$$= \Lambda^\mu_\rho p^\rho \Lambda^\sigma_\mu p_\sigma U(\Lambda)\Psi$$

$$= p^\mu p_\mu U(\Lambda)\Psi$$

$$\Updownarrow$$

$$U(\Lambda)\hat{P}^\mu \hat{P}_\mu \Psi = p^\mu p_\mu U(\Lambda)\Psi$$

$$\Rightarrow [\hat{P}_\mu \hat{P}^\mu, U(\Lambda,0)] = 0$$

The momentum operators are invariant under translations, that is, we will have

$$[\hat{P}_\mu \hat{P}^\mu, U(1,a)] = 0.$$

$$\Rightarrow [\hat{P}_\mu \hat{P}^\mu, U(\Lambda,a)] = [\hat{P}_\mu \hat{P}^\mu, U(\Lambda,0)U(1,a)]$$

$$= [\hat{P}_\mu \hat{P}^\mu, U(\Lambda,0)]U(1,a) + U(\Lambda,0)[\hat{P}_\mu \hat{P}^\mu, U(1,a)]$$

$$= \boxed{0}$$