

Weinberg
2.4

$$\hat{P}^M U(\Lambda) \Psi = \Lambda^M_{\rho} p^{\rho} U(\Lambda) \Psi \quad (\text{Weinberg 2.5.2})$$

$$\begin{aligned} \Rightarrow \hat{P}_{\mu} \hat{P}^M U(\Lambda) \Psi &= \Lambda^M_{\rho} p^{\rho} \hat{P}_{\mu} U(\Lambda) \Psi \\ &= \Lambda^M_{\rho} p^{\rho} \Lambda^{\sigma}_{\mu} p_{\sigma} U(\Lambda) \Psi \end{aligned}$$

$$= p^M p_{\mu} U(\Lambda) \Psi$$



$$U(\Lambda) \hat{P}^M \hat{P}_{\mu} \Psi = p^M p_{\mu} U(\Lambda) \Psi$$

$$\Rightarrow [\hat{P}_{\mu} \hat{P}^M, U(\Lambda, 0)] = 0$$

The momentum operators are invariant under translations, that is, we will have

$$[\hat{P}_{\mu} \hat{P}^M, U(1, a)] = 0.$$

$$\begin{aligned} \Rightarrow [\hat{P}_{\mu} \hat{P}^M, U(\Lambda, a)] &= [\hat{P}_{\mu} \hat{P}^M, U(\Lambda, 0) U(1, a)] \\ &= [\hat{P}_{\mu} \hat{P}^M, U(\Lambda, 0)] U(1, a) + U(\Lambda, 0) [\hat{P}_{\mu} \hat{P}^M, U(1, a)] \\ &= \boxed{0} \end{aligned}$$